5.3 It’s All In Your Head

A Solidify Understanding Task

In the previous task you were asked to justify some claims by writing paragraphs explaining how various figures were constructed and how those constructions convinced you that the claims were true. Perhaps you found it difficult to say everything you felt you just knew. Sometimes we all find it difficult to explain our ideas and to get those ideas out of our heads and written down or paper.

Organizing ideas and breaking complex relationships down into smaller chunks can make the task of proving a claim more manageable. One way to do this is to use a flow diagram.

First, some definitions:

- In a triangle, an **altitude** is a line segment drawn from a vertex perpendicular to the opposite side (or an extension of the opposite side).

- In a triangle, a **median** is a line segment drawn from a vertex to the midpoint of the opposite side.

- In a triangle, an **angle bisector** is a line segment or ray drawn from a vertex that cuts the angle in half.

- In a triangle, a **perpendicular bisector of a side** is a line drawn perpendicular to a side of the triangle through its midpoint.

Travis used a compass and straightedge to construct an equilateral triangle. He then folded his diagram across the two points of intersection of the circles to construct a line of reflection. Travis, Tehani, Carlos and Clarita are trying to decide what to name the line segment from C to D.

Travis thinks the line segment they have constructed is also a median of the equilateral triangle. Tehani thinks it is an angle bisector. Clarita thinks it is an altitude and Carlos thinks it is a perpendicular bisector of the opposite side. The four friends are trying to convince each other that they are right.
Here is a flow diagram of statements that can be written to describe relationships in the diagram, or conclusions that can be made by connecting multiple ideas.

Use four different colors to identify the statements each of the students—Travis, Tehani, Carlos and Clarita might use to make their case.

Given: $\triangle ABC$ is equilateral

- $AB \equiv BC \equiv AC$
- $\angle CDA$ and $\angle CDB$ are right angles

$\overrightarrow{CE}$ is a line of reflection

- $\overrightarrow{CD} \perp \overrightarrow{AB}$
- $D$ is the midpoint of $AB$
- $AD \equiv DB$
- $\overrightarrow{CD} \cong \overrightarrow{CD}$

$\angle ACD \cong \angle BCD$  \hspace{1cm} $\triangle ACD \cong \triangle BCD$

therefore, $\overrightarrow{CD}$ is an altitude

therefore, $\overrightarrow{CD}$ is a median

therefore, $\overrightarrow{CD}$ is an angle bisector

therefore, $\overrightarrow{CD}$ is a perpendicular bisector
Match each of the arrows and braces in the flow diagram with one of the following reasons that justifies why you can make the connection between the statement (or statements) previously accepted as true and the conclusion that follows:

1. Definition of reflection
2. Definition of translation
3. Definition of rotation
4. Definition of an equilateral triangle
5. Definition of perpendicular
6. Definition of midpoint
7. Definition of altitude
8. Definition of median
9. Definition of angle bisector
10. Definition of perpendicular bisector
11. Equilateral triangles can be folded onto themselves about a line of reflection
12. Equilateral triangles can be rotated 60° onto themselves
13. SSS triangle congruence criteria
14. SAS triangle congruence criteria
15. ASA triangle congruence criteria
16. Corresponding parts of congruent triangles are congruent

Travis and his friends have seen their teacher write two-column proofs in which the reasons justifying a statement are written next to the statement being made. Travis decides to turn his argument into a two column proof, as follows.

<table>
<thead>
<tr>
<th>Statements</th>
<th>Reasons</th>
</tr>
</thead>
<tbody>
<tr>
<td>ΔABC is equilateral</td>
<td>Given</td>
</tr>
<tr>
<td>(\overline{CE}) is a line of reflection</td>
<td>Equilateral triangles can be folded onto themselves about a line of reflection</td>
</tr>
<tr>
<td>(D) is the midpoint of (\overline{AB})</td>
<td>Definition of reflection</td>
</tr>
<tr>
<td>(\overline{CD}) is a median</td>
<td>Definition of median</td>
</tr>
</tbody>
</table>

Write each of Tehani’s, Carlos’, and Clarita’s arguments in two-column proof format.